Undersampling to accelerate computations

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Abstract. *Undersampling means overcoming the Nyquist threshold. Modern Compressive Sensing theory provides general undersampling approach for data which can be represented in some domain in sparse or compressible form. Computationally complex problems frequently benefit in performance from Fourier decomposition based methods. Solution of such problems can be accelerated by decreasing the sampling rate. Undersampling application to accelerate finite-difference computations of synthetic datasets for 3D seismic surveys in elastic anisotropic geology medium with thin-layered structures and complex fracturing is described as a sample.*

Keywords

Compressive Sensing, undersampling, full-wave seismic modeling, GPU programming.

1 Introduction

In recent years, a new theory, which suggest that it may be possible to surpass the traditional limits of sampling density inspired more than a thousand papers and pulled in millions of dollars in both international and national research grants. The theory called Compressive Sensing (CS) has attracted considerable attention in applied mathematics, computer science, physics, chemistry, biology, medicine, engineering and geosciences by suggesting. CS builds upon the fundamental fact that we can represent useful signals by just a few non-zero coefficients in a suitable basis or dictionary.

The theoretical foundation of CS is based on pioneering work of Emmanuel Candès, Terrence Tao, Justin Romberg [1, 2], David Donoho [3, 4], who showed that a finite-dimensional signal having a sparse or compressible representation can be recovered from a small set of linear, nonadaptive measurements. (While this idea has only recently gained significant attraction it was first proposed in 1795 [5], re-invented in early 1900's [6] and developed by modern researchers since late 1990's [7-9].)

Considerable efforts have been devoted in recent years by many researchers to adapt the theory of CS to better solve real-world challenges. CS-based hardware listed in [10] targets wide range of applications from curious devices such as "one-pixel camera" to wideband analog signal receivers, high-speed MRI scanners and high-throughput screening, reduced-cost seismic imaging, X-Ray astronomy camera and ground penetrating radar, audio, video, radio receivers, etc. The most obvious CS applications are related to completing data recorded in unfavorable conditions [11] or to decrease sensing expenses. But CS can be useful tool also for acceleration of many computationally hard problems. Let discuss an example of geophysical simulation application which uses CS for calculation acceleration purpose.

2 Method of 2.5D Full Wave Modeling for Synthesis of 3D Seismograms

Synthetic seismograms calculated for a given geological model and survey geometry allow one to estimate the practical effect of more expensive prospecting in specific geological conditions. The most common approach for the numerical solution of the Full-Wave Modeling (FWM) problem is the explicit finite difference (FD) method [12]. Since wide source-receiver offsets and deep penetration of typical seismic acquisition the simulation grid dimensions are measured in hundreds of wavelengths. Preservation of wavelet requires about 10 grid cells per wavelength. As result, the 3D FD simulation grid consists of billions of nodes. Given the 3D anisotropic elastic FWM must keep in memory 31 array of at least single-precision numbers (of length 4 bytes) each iteration needs to process hundreds of gigabytes. This value exceeds RAM volume of the majority of modern cluster nodes and multiprocessor servers.

In spite of the long history and diversity of approaches [13, 14] the 3D FWM, until recently, was mostly the field of scientific research. Now there is an opportunity to simulate wavefield propagation within acceptable time due to the progress in high-performance computing, especially on general-purpose graphics cards (GPU). For example, [15] reported reaching 25-50 times acceleration of the anisotropic elastic modeling on NVIDIA GPU-cluster. However, the computational cost of modeling a full set of synthetic 3D seismic data (which includes thousands of wavefields) still substantially exceeds the practical limitations. E.g., SEAM II 3D FWM project [16] run by powerful business/science consortia recently used the cluster Sierra (1944 x 12 CPU cores) for year to generate a single synthetic dataset (each of thousands wavefields was simulated for 13 hours by 3840 cores).

3D FWM can be simplified, if all properties of the medium are fixed along one direction (Y). Models of such type (figure 1) are called "two and a half dimensional" (2.5D). Limitation of 2.5D makes it impossible to synthesize seismograms corresponding to real objects. Consequently, 2.5D does not help to optimize survey design in terms of target horizon illumination. However, 2.5D is useful to investigate the influence of multi-component acquisition and density of survey on interpretation of complex media. 2.5D FWM accounts thin-layering, fracturing and arbitrary anisotropy of models too complex for ray tracing.

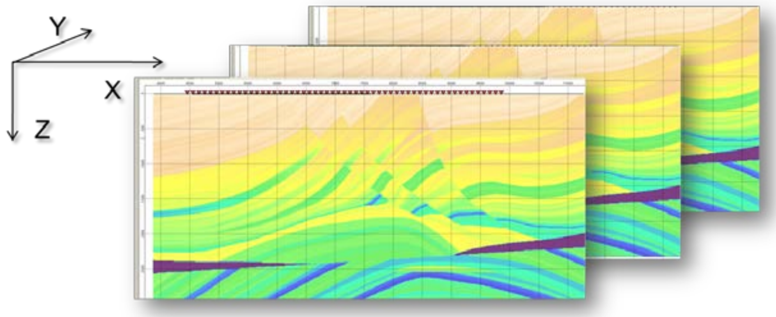

Fig. 1. 2.5D Model

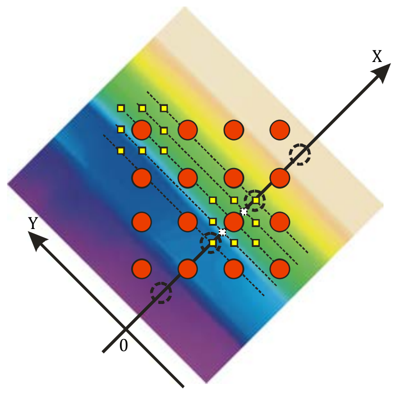

Fig. 2. Source merge for 2.5D

3D elastic anisotropic system of equations expresses Hooke's law combined with Newton's 2nd law:

$$\begin{cases} \rho(X)\dfrac{\partial u_n(X,t)}{\partial t} = \dfrac{\partial \tau_{nk}(X,t)}{\partial x_k} + f_n(X,t) \\ \dfrac{\partial \tau_{mn}(X,t)}{\partial t} = \Lambda_{mnkq}(X)\dfrac{\partial u_k(X,t)}{\partial x_q} + \dfrac{\partial M_{mn}(X,t)}{\partial t}, \end{cases} \tag{1}$$

where $X = (x_1 \quad x_2 \quad x_3)$ means a point in 3D space, u_n denote displacement velocities, τ_{nm} are components of a stress tensor. Parameters of the geology media are stiffness tensor Λ and density ρ. Source signal is encoded by the vector functions of source forces f_n and moment forces M_{nm}. Both type forces are zero out of source.

In 2.5D case $\Lambda(x,y,z) = \Lambda(x,0,z)$ and $\rho(x,y,z) = \rho(x,0,z)$ for arbitrary y. So it is convenient to represent the wavefield in Fourier domain by decomposition (1) along the axis x_2:

$$u_n(X,t) = \int_{-\infty}^{+\infty} \tilde{u}_n(\tilde{X},t)e^{i\omega y}dy, \quad \tau_{mn}(X,t) = \int_{-\infty}^{+\infty} \tilde{\tau}_{mn}(\tilde{X},t)e^{i\omega y}dy, \tag{2}$$

where $\tilde{X} = (x_1 \quad \omega \quad x_3)$. The system (2) can be re-written in the space Fourier domain ($q \in \{1,3\}$):

$$\begin{cases} \rho(\tilde{X})\dfrac{\partial \tilde{u}_n(\tilde{X},t)}{\partial t} = \dfrac{\partial \tilde{\tau}_{nq}(\tilde{X},t)}{\partial x_q} + i\omega\tilde{\tau}_{n2}(\tilde{X},t) + \tilde{f}_n(\tilde{X},t) \\ \dfrac{\partial \tilde{\tau}_{mn}(\tilde{X},t)}{\partial t} = \Lambda_{mnkq}(X)\dfrac{\partial \tilde{u}_k(\tilde{X},t)}{\partial x_q} + \Lambda_{mnk2}(X)i\omega\tilde{u}_2(\tilde{X},t) + \dfrac{\partial \tilde{M}_{mn}(\tilde{X},t)}{\partial t} \end{cases} \tag{3}$$

Song and Williamson [17] first time used similar technique for the 2.5D-acoustic approximation of constant density models. 2.5D FWM for arbitrary 3D TTI anisotropy and fracturing was described in [18] and [19].

Since seismogram is not changed if both the sources and receivers similarly shift along the axis Y, in 2.5D model it is sufficient to simulate only a single shot line. Other shot lines are easy generated by copying traces with change of coordinates in their headers. As result, in the case of 2.5D model the computations can be reduced by tens, sometimes hundreds times, while all the 3D dataset features persist for further processing, see figure 2.

The equations (3) look very similar to (1). However, $\tilde{u}_n$ and $\tilde{\tau}_{mn}$ are complex numbers when u_n and τ_{mn} are real. As result implementation of the system (3) requires approximately two times more calculations per cell. At the same time, $\tilde{u}_n(x_1 \quad -\omega \quad x_3)$ is complex conjugate to $\tilde{u}_n(x_1 \quad \omega \quad x_3)$, and $\tilde{\tau}_n(x_1 \quad -\omega \quad x_3)$ is complex conjugate to $\tilde{\tau}_n(x_1 \quad \omega \quad x_3)$. Hence we don't need to simulate wave propagation for negative frequencies. Thus calculations for one pair of symmetric frequencies ω and $-\omega$ by (3) require approximately the same number of operations and expands approximately the same time as (1) for each $y = const$. (The spatial/time samplings are supposed to be equal as determined by stability and dispersion conditions.) So acceleration rate due to 2.5D can be estimated as a ratio of the grid size in y direction to the number of non-negative frequencies ω in the wavefield discrete Fourier transform. Using Nyquist frequency for reconstruction one can expect the next computational complexity ratio [20]:

$$Speedup = \frac{N_y}{N_\omega} \approx W \Big/ \left(\frac{1}{2} + \frac{V_{\max} t_{\max}}{y_{\max} - y_{\min}} \right). \tag{4}$$

3 Undersampling for Acceleration of 2.5D Simulation Method

Synthetic seismogram compressibility in time/space domain [21] creates conditions for use of CS approach to simulate less quasy-2D problems. First we have applied irregular (either random or jittered) undersampling in combination with the stepwise thresholding described in [11]. Then we improved the method by adding interpolation of missed space frequencies. As result we have sampled (simulated) 4 times below the Nyquist rate for the cost of 1% noise (figure 3).

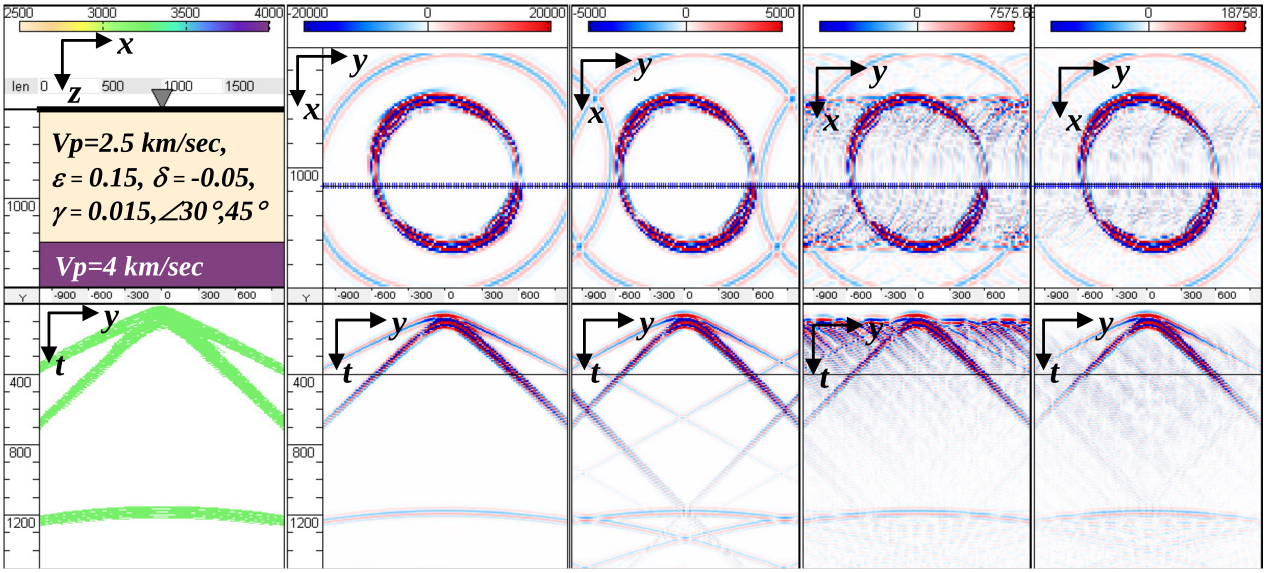

Fig. 3. Model and signal mask (left column), correct sampling, regular undersampling, random undersampling, random undersampling with spectrum interpolation (right column)

Regardless of use linear or trigonometric interpolation method the tests demonstrated about 10 times better Signal-to-Noise Ratio (SNR) than without interpolation. Besides, we have found that effect of missed/interpolated images is smaller for higher frequencies, until probability misbalance is small. This fact can be explained by more stable (continues) low frequency part of seismogram space spectra.

4 Conclusion

2.5D reduces a 3D problem to a set of independent quasi-2D problems and uses GPU in the most favourable mode: without data reload, using single floating point precision and local calculations only. Because of the synergy both high level and low level GPUs demonstrate excelent performance of 2.5D (up to 80-120 times higher than a serial CPU program [22]) despite unfavourable estimate (4).

Additional 3-4 times acceleration of 2.5D over full 3D can be obtained for simple models by random undersampling with spectrum interpolation (for the cost of about 1% noise). A 2.5D analog of a SEAM II shotgather can be calculated for less then two days by an average GPU equipped PC, and due to tens to hundred times reduced shot number the whole job can be executed on a PC instead of supercomputer. Because of multiple quasy-2D sub-problems per shot the method is perfectly scalable for GPU clusters.

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