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Analysis of approximation algorithms computations parallelizations

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Abstract. *In this work the known tasks of the schedules for the works connected with relation of the partial order are studied. Mathematical models of the consistent relations are the oriented graphs. The tasks are formulated as optimization tasks on graphs. Since they belong to the class of NP-complete problems, it is appropriate the search of effective approximate algorithms with polynomial complexity. The authors identify sub-classes of graphs studying well-known algorithms and propose new ones. In particular, the following graphs are researched: unbranched arithmetic expressions and branched arithmetic expressions, and also cyclic processes were treated. It is shown that the estimate of the algorithm accuracy based on the level principle is achievable even for graphs that model unbranched arithmetic expressions.*

Keywords

Parallel scheduling, length and width of scheduling, multiprocessor computer systems.

1 Introduction

The tasks of the parallel scheduling studied in this work are still insufficiently investigated. There are the exact algorithms for their solution in the general case with exponential complexity (branch and bound methods, the method of reducing the problem to the integer linear programming, method of reducing the problem to maximum conditional flow problem). However, application of these methods for large-scale practical problems is impossible due to high computational complexity. Therefore, it is advisable to allocate either subclasses of graphs for which it is possible to get exact algorithms of polynomial complexity, or to develop approximate algorithms that give acceptable results for practice.

2 Main sections

A lot of large-scale applied problems, which require solution in the real-time mode, can be solved only in case of the process optimal organization.

Among such problems some deserve special attention. Separate stages or operations of these problems are bounded with a condition of the non-contradictory consecution.

The mathematical model of such relationships is presented by oriented acyclic graphs in which the vertices correspond to specific operations, and two vertices are connected by an arc when one operation immediately precedes the other.

For the first time such approach was suggested in the work [1]. The tree that models a process of the assembly on the assembly line was studied there.

Two most researched mutually dual optimization problems are as follows:

1. Given: a finite partially ordered set of operations and a finite number of workers.
   Required: to distribute operations among the workers so that all works are completed in a minimal time without violations of the partial order.
2. Given: a finite partially ordered set of operations and the completion time.

Required: to determine the minimum number of workers that will complete all operations in a given time without violations of the partial order.

The seeming simplicity of the formulated problems draws the researchers' attention.

In reality, these problems belong to the class of NP-complete problems for which there is a hypothesis that exact algorithms of polynomial complexity for their solution don't exist.

Therefore, the following research directions are of interest: developing of the exact algorithms of polynomial complexity for special subclasses of graphs, for special subtasks; developing of the effective approached algorithms with an aprioristic estimation of the solution quality.

Problems stated above are studied by professionals in scheduling theory, by researchers studying the parallelization of computations, by programmers developing practical software systems [2].

The problem statements illustrated above can be formulated in terms of optimization problems using graphs.

In order to do this, let's introduce definitions.

Definition1. Linear scheduler of elements S of a finite set V (|V | = n) is called the placement of these elements on n locations arranged in a line, each element has only one location.

Definition2. Length ℓ of the scheduler S is the number of non-empty locations in the scheduler.

Definition3. Width h of the scheduler S is the quantity h(S) = $\max_i$ |S[i]|, where i=1,...,n; S[i] – the set of elements, that are placed on the location i in the scheduling S.

Let's consider the oriented acyclic graph G (V, U).

Definition4. Parallel scheduler of the vertices of a digraph G is such linear scheduler of its vertices that if the pair (i,j) $\in$U, then the vertex i is located to the left from the vertex j in the scheduler S, that is if (i,j) $\in$U and (i$\in$S[p], j$\in$S[q]) then p<q.

Then we can introduce the mathematical model of tasks 1 and 2 using graphs.

1. Given: a graph G and a scheduler width h.
   Required: to find a parallel scheduler of the minimum length and given width.
2. Given: a graph G and a scheduler length ℓ.
   Required: to find a parallel scheduler of the minimum width and given length.

When de-parallelizing parallel computations, the graph specifying the order of operations is not arbitrary, and has a certain structure. If the algorithm describes the computing procedure corresponding to non-branched arithmetic operations, then the graph is such that each vertex has not more than two incoming arcs.

When modeling branched arithmetic operations that are characterized by condition "IF", certain sub graphs of the graph should be de-parallelized.

When de-parallelizing the loop parts of the code it is not always enough to de-parallelize the loop section of the code and to take into account the number of its executions.

The results of using the algorithms based on the level principle and the level principle with the lexicographical marking of vertices have been studied. And new combined algorithms for the solution of problem 1 were proposed.
It is known that the algorithm based on the level principle is accurate for incoming trees and forests.
For arbitrary graphs, this algorithm gives an approximation with an estimate of the quality $\frac{\ell_A(S)}{\ell(S^*)} \le \frac{2h}{h+1}$, wherein $\ell_A(S)$ and $\ell(S^*)$ are respectively lengths of the scheduler obtained by the algorithm and the optimal one.
The use of this algorithm for graphs describing non-branching arithmetic expressions produces the same quality solutions, and this estimate is practically achievable.

The graph setting the order of the operation execution is on the first picture (Pic. 1). The results for this graph and height 4 are as follows:

1. The branches and bounds algorithm (an exact algorithm) and the new combined algorithm:

$$S^* = < \begin{matrix} 1 & 2 & 7 & 12 & 17 \\ 3 & 6 & 9 & 13 & 18 \\ 4 & 8 & 11 & 14 & 19 \\ 5 & 10 & 15 & 16 & 20 \end{matrix} >;$$

$$\ell(S^*) = 5.$$

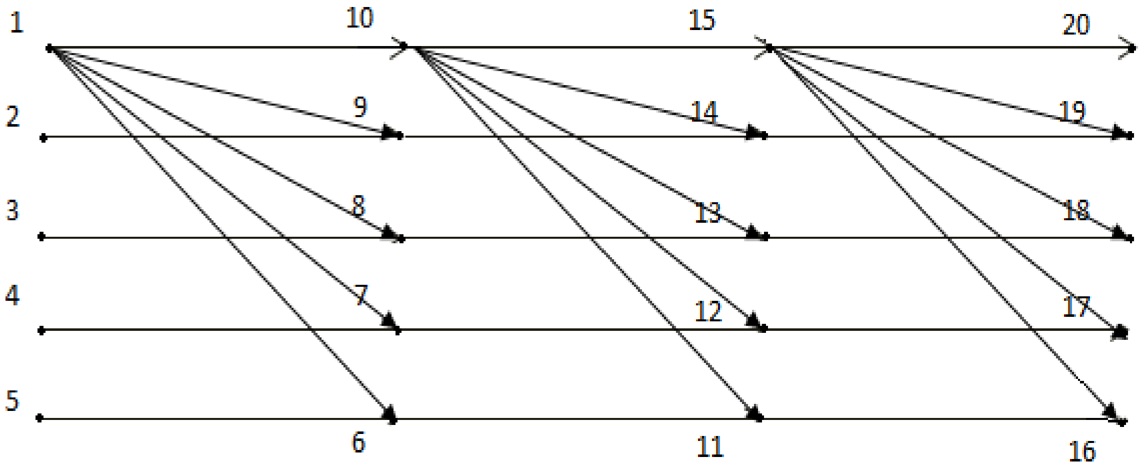
2. The algorithm based on the level principle:

$$S = \left\langle \begin{array}{cccccccc} 1 & & 6 & & 11 & & 16 & \\ 2 & & 7 & & 12 & & 17 & \\ 3 & 5 & 8 & 10 & 13 & 15 & 18 & 20 \\ 4 & & 9 & & 14 & & 19 & \end{array} \right\rangle;$$

$$\ell_A(S) = 8.$$

Therefore, it is not appropriate to apply the algorithm based on the level principle for de-parallelization of real-world computing. The most useful results in terms of accuracy are given by the composite algorithms.

The computational experiment was held to make a comparison of the algorithms for solving the real tasks. During the experiment, the random acyclic oriented graphs were generated and for them schedulers were built by different algorithms. The same was made for graphs modeling non-branched arithmetic operations. As a result, it was shown that the combined algorithms give the best results. In more than 70% cases, the results of the combined algorithm are the same as the results of the branches and bounds method (exact solution).



Pic. 1. Presents the graph.

3 Conclusion

In this work, it was shown that the combined algorithms should be used for the large-scale tasks. Besides, in future researches, it's appropriate to study sub-classes of graphs modeling the real processes and to develop exact polynomial algorithms for solving scheduling tasks on such graphs.

References

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